

Application of Operational Transfer Path Analysis on a Classic Car

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Abstract

This paper discusses the analysis of a classic car with the Operational Transfer Path Analysis (OTPA). This method poses a fast and efficient way of identifying critical vibration paths and source contributions within vehicle design. Only measurement data of the operating vehicle are needed to perform this analysis. Real measurement situations for example are several vehicle run-ups, full load, partial load and coastdown. These kinds of measurements are commonly performed in the automotive industry and can therefore be easily adopted. The method OTPA is introduced and discussed based on a classic car Wartburg 311. This vehicle is described in the project sketch “method comparison of the transfer path analysis” (Fraunhofer IWU Dresden) and provided by Westsächsische Hochschule Zwickau, Institut für Kfz-Technik (IfK).

Introduction

OTPA uses a signal processing method which finds the linearized Transfer Functions (TF) between a set of chosen input and output channels from a measurement. The in- and output relations are determined such, that they are independent quantities. The resulting Transfer Functions can be used in a Transfer Path Analysis (TPA), determining a source’s propagation of noise and its resulting contribution in the output (response) signal [2]. In practice, numerical operations involved could suffer from bad conditioning. Use is therefore made of the Singular Value Decomposition (SVD) technique to prevent such problems. Indeed, it was found that rejecting smaller singular values is necessary to reduce measurement noise and other unwanted influences on the transfer function estimates.

Theory

Consider an arbitrary linear(ized) system described by a set of input and output DoF, represented as

$$\mathbf{H}(j\omega)\mathbf{x}(j\omega) = \mathbf{y}(j\omega). \quad (1)$$

Here $\mathbf{H}(j\omega)$ is the Transfer Function matrix linking the vector of input DoF $\mathbf{x}(j\omega)$ to the vector of output DoF $\mathbf{y}(j\omega)$. The dependency on frequency is denoted by $(j\omega)$. In NVH problems, the measured signals are typically motions, denoted $\mathbf{u}(j\omega)$, forces $\mathbf{f}(j\omega)$ and sound pressures $\mathbf{p}(j\omega)$. The input and output vectors

can thus in general be assembled from these quantities as

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \mathbf{u}_x \\ \mathbf{f}_x \\ \mathbf{p}_x \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \mathbf{u}_y \\ \mathbf{f}_y \\ \mathbf{p}_y \end{bmatrix} \quad (2) \\ \mathbf{u}_\star &= [u_\star^{(1)} \dots u_\star^{(k)}]^T \\ \mathbf{f}_\star &= [f_\star^{(1)} \dots f_\star^{(l)}]^T \\ \mathbf{p}_\star &= [p_\star^{(1)} \dots p_\star^{(m)}]^T, \end{aligned}$$

where the dependency on frequency is omitted for clarity.¹ Notice that it is up to the engineer to define the input and output sets from the measured data. He is not restricted to choose forces as excitations only, but motion and sound pressure, which are responses from a physical point of view, may also be chosen. This makes the OTPA method a true engineering tool, requiring appropriate setup of the experiment. Done well, OTPA has shown to be very fast and accurate, enabling cost efficient product development.

Not all physical quantities have to be present in each set defined in (2), neither do the vectors have to have the same dimensions. In fact, usually the number of excitations, given in the input vector, will be larger than the number of DoF in the response vector. In vehicle analysis a typical example is to find the transfer functions between motions measured on the driveline and the sound pressure at the driver’s ear.

By the construction of (1), the elements of the TF matrix \mathbf{H} have the form

$$H_{ij} = \left. \frac{y_i}{x_j} \right|_{x_k=0}; \quad k \neq j. \quad (3)$$

Typically this property is used in Experimental Modal Analysis (EMA), where an external force f is applied, by shaker or impulse hammer, as the only input DoF and the resulting responses \mathbf{u} of the system are chosen as outputs. These kind of Transfer Functions are denoted *receptance* Frequency Response Function (FRF) in the literature and have the special property that their frequency peaks show the free system’s eigenfrequencies. Strictly, one

¹It is also allowed to use any other physical quantities, as long as they are synchronized during measurement with the other quantities.

could thus determine a column of the TF matrix in OTPA also by exciting the system with only the input DoF x_j , while suppressing all other input excitations. In practice this is very hard to achieve as inputs are not only forces, but also motions, sound pressures or any kind of quantities. The determination of the transfer functions in this way will therefore often lead to very difficult, impractical and sometimes even impossible experimental setups. Analysis as such will therefore require a big expense in time and resources.

To overcome this disadvantage, the OTPA method determines all elements of the TF matrix from *one* measurement only where all excitations are present at once. This determination is discussed next by first taking the transpose of equation (1) and writing the equation on entry level:

$$\begin{bmatrix} x^{(1)} & \dots & x^{(m)} \end{bmatrix} \begin{bmatrix} H_{11} & \dots & H_{n1} \\ \vdots & \ddots & \vdots \\ H_{1m} & \dots & H_{mn} \end{bmatrix} = \begin{bmatrix} y^{(1)} & \dots & y^{(n)} \end{bmatrix}. \quad (4)$$

Here m and n denote the number of in- and output DoF. Taking the transpose does not allow the determination of the FRF elements though. In order to do so, notice that during an operational measurement of, for example, a vehicle run-up on a dynamometer, a set of synchronized measurement blocks will be collected. In general these sets will not have the same content, as the excitations change continuously during the measurement. If one requires, or defines, the relation between the input and output DoF as being linear(ized) and constant during the total measurement, equation (4) should however hold for each individual measurement block. One could thus extend equation (4) writing the equation for all measurement blocks r , yielding:

$$\begin{bmatrix} x_1^{(1)} & \dots & x_1^{(m)} \\ \vdots & \ddots & \vdots \\ x_r^{(1)} & \dots & x_r^{(m)} \end{bmatrix} \begin{bmatrix} H_{11} & \dots & H_{n1} \\ \vdots & \ddots & \vdots \\ H_{1m} & \dots & H_{mn} \end{bmatrix} = \begin{bmatrix} y_1^{(1)} & \dots & y_1^{(n)} \\ \vdots & \ddots & \vdots \\ y_r^{(1)} & \dots & y_r^{(n)} \end{bmatrix}. \quad (5)$$

or

$$\mathbf{X}\mathbf{H} = \mathbf{Y}, \quad (6)$$

written more compact in matrices format.

The explicit determination of the Transfer Function matrix \mathbf{H} by (6) cause numerical problems in the inversion of the term \mathbf{X} though. Use is therefore made of the Singular Value Decomposition (SVD) technique,

to overcome this kind of problems. Indeed matrix \mathbf{X} can be replaced by the Singular Value Decomposition as

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T. \quad (7)$$

Here \mathbf{U} is an $r \times m$ unitary matrix, $\mathbf{\Sigma}$ is a $m \times m$ rectangular matrix with nonnegative numbers on the diagonal and zeros off the diagonal. \mathbf{V}^T denotes the conjugate transpose of \mathbf{V} , an $m \times m$ unitary matrix. The Singular Value Decomposition of \mathbf{X} can be directly used in the computation of the FRF matrix \mathbf{H} yielding

$$\mathbf{H} = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{Y}. \quad (8)$$

where $\mathbf{\Sigma}^{-1}$ is the inverse of $\mathbf{\Sigma}$. It was found that smaller singular values are caused by noise influences and other external disturbances [1]. They are therefore unwanted and need to be rejected to obtain proper TF estimates.

Application

This section discusses an OTPA analysis on a classic vehicle, type Wartburg 311, see figure 1. The vehicle



Figure 1: Measured Wartburg 311 on a vehicle dynamometer.

was measured with multiple sensors as listed, along with other details, in table 1. Any OTPA analysis first starts with a comparison of synthesized outputs with their actual measured output values from the measurement. Indeed, after the Transfer Functions are determined from (8) using them as FIR filters in equation (6), the output channels \mathbf{Y} can be calculated / synthesized with the measured inputs \mathbf{X} as well. Comparing them with the actual measured outputs shows one how well the identified sources (/ used input signals) model the actual system's response. Differences between measured and synthesized outputs therefore shows one if there are other (incoherent) sources in play, which didn't take part in the modeling. Figure 2 shows this comparison for a WOT3 measurement of the Wartburg 311. Note that here all Auto Power Spectrum (APS) measurement blocks of the complete runup were averaged to get an overall indication of the model's fit. As seen in the figure the Wartburg could be modeled well, using all transition points from table 1, up to a frequency of about 1200Hz. At higher frequencies differences get into play, which

Vehicle Properties

Type	<i>Wartburg 311</i>
Motor	<i>3 cyl. 900 cm³ two-stroke</i>
Performance	<i>28 kW (37hp) at 4000 rpm</i>
Top speed	<i>115 km/h</i>
Fuel consumption	<i>9,8 l/100km</i>
Transmission	<i>4-speed front wheel drive</i>

Dynamometer Properties

Type	<i>48 inch diameter with hydro break</i>
Surface	<i>Road</i>
Transmission	<i>2 wheel drive</i>

OTPA Setup

Sources	<i>Engine & Front Tires</i>
Transition (SB)	<i>Engine mounts (1x) Exhaust (2x) Gearbox (2x) Front wheels (2x)</i>
Transition (AB)	<i>Intake orifice(1x) Engine compartment (2x) Exhaust orifice (1x)</i>
Receiver	<i>Driver's ear (1x)</i>

Measurement Settings

Sampling rate	<i>8192 Hz</i>
Block length	<i>0,5 sec</i>
Run-up type	<i>2x WOT3 2x WOT4 2x POT3 2x POT4</i>

Table 1: Measurement details. “SB” denotes “Structure Borne” and therefore in this case measured accelerations. “AB” denotes “Airborne” and therefore in this case measured sound pressures. “WOT” denotes “Wide Open Throttle”; “POT” denotes “Partial Open Throttle” and the number the chosen gear respectively.

assumably originate from additional airborne contents of for example the tires which were not measured.

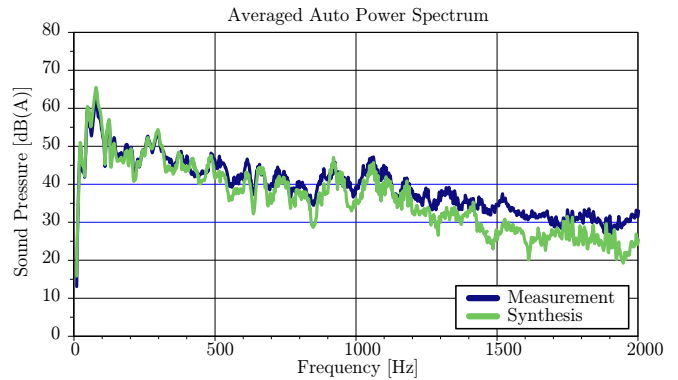


Figure 2: Measured Wartburg 311 on a vehicle dynamometer.

Secondly the main path contributions² are analyzed as shown in figure 3. The figure shows the engine’s intake has a high average contribution between 60 - 100 Hz and 400 - 500 Hz. The attachment of the exhaust at the rear end has a high average contribution between 100 - 200 Hz. From this path contribution analysis one can therefore determine which components are most worthwhile improving. However, although

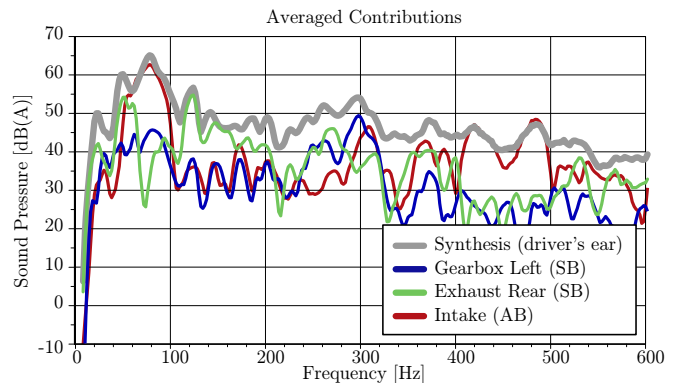


Figure 3: Averaged path contributions of the vehicle from the Wartburg’s WOT3.

figure 3 shows the overall component performances, figure 4 coupled with figure 5 can also be adopted to show path contributions in individual operating conditions. This is indeed typically a third step of an OTPA analysis. Here specific product details are analyzed, where deflection shapes, phase information and sound modification can be adopted to come up with product improvements.

If one, for example, decouples the attachment of the exhaust at the rear and remeasures the Wartburg 311 in a WOT3 measurement, a clear reduction in sound level of the second engine order is seen. Using the OTPA method, this reduction was also predicted from the measurement data of the original vehicle data. Figure 6 shows the modified vehicle measurement and the

²The sum of all path contributions yield the synthesized output signal itself. One path contribution is therefore the signal content of that single input / source contained in the synthesized output signal.

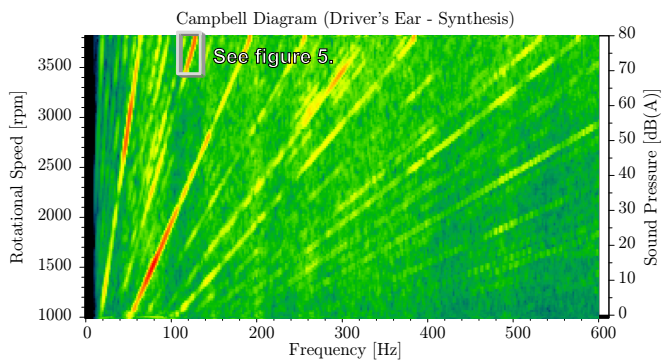


Figure 4: Campbell diagram of the synthesized sound pressure at the driver's ear from the OTPA analysis.

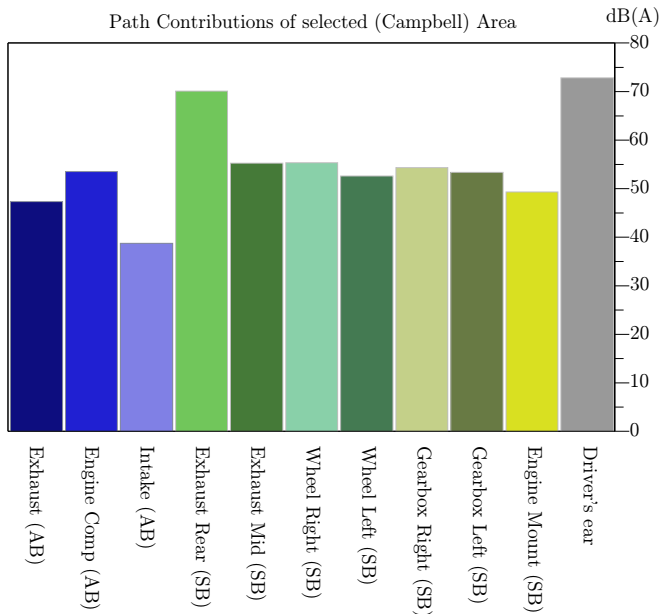


Figure 5: The individual bars show the main path contributions of the selected area in figure's 4 Campbell Diagram.

simulated OTPA results match well, illustrating the OTPA's potential to simulate vehicle modifications.

Conclusion

This paper illustrates, using a dynamometer measurement of the classic Wartburg 311, the strength of Operational Transfer Path Analysis. The analysis shows that especially the engine's intake and exhaust are dominating the sound level inside the driver's compartment.

References

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- [2] D. de Klerk, Dynamic Response Characterization of Complex Systems through Operational Identification and Dynamic Substructuring, PhD. Thesis, Delft University of Technology, **March** (2009)

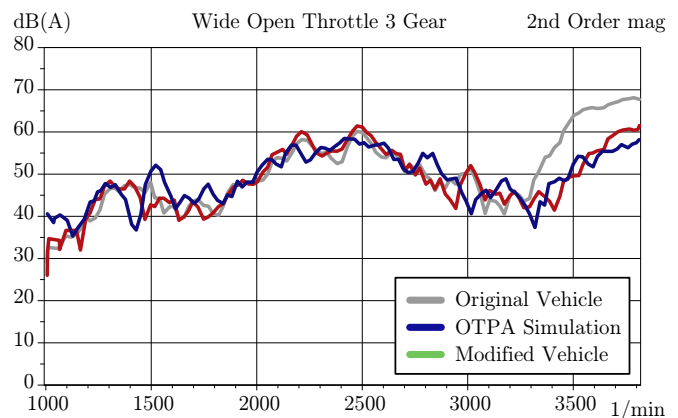


Figure 6: The attachment of the exhaust at the rear has a dominant influence on the second engine order. After a renewed measurement of the Wartburg 311 with disconnected exhaust, a clear reduction is visible. Using OTPA the reduction was also predicted from the original vehicle measurement. Good agreement is seen between the OTPA simulation and the measurement of the modified vehicle.